

An Efficient Dual-Observer Method for Leader-Following Consensus Control of Multiagent Systems

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Abstract—Many existing studies on multi-agent systems (MASs) control have primarily focused on output or consensus tracking problems, often neglecting the impact of external disturbances and the efficient use of control resources. In this paper, we propose a novel adaptive dual-observer control scheme that implements a switched-threshold event-triggered strategy to optimize communication resource utilization. A neural network is leveraged to approximate the nonlinear dynamics of MASs, and the adaptive control strategy is designed to achieve consensus tracking. Then we provide stability analysis to ensure that each agent's output can effectively follow the leader's trajectory with a controllable bounded tracking error. Additionally, our scheme is capable to pre-select the lower bound of triggering intervals, which prevents Zeno behaviors, a critical aspect for the practical implementation of controllers. Finally, simulation results on a radar transmitter formation problem validate the effectiveness of our proposed approach.

Index Terms—Observer design, event-triggered control, consensus tracking, multi-agent systems, nonlinear control

I. INTRODUCTION

In the past two decades, the study of multiagent systems (MASs) has witnessed considerable growth, driven by their extensive applications in various application scenarios [1]–[4]. These systems provide effective solutions for coordination and control in complex environments. This paper specifically addresses the challenge of signal consensus tracking of radar transmitter formation within the MASs framework. In modern vehicular systems [5], radar signal transmitters play a crucial role, facilitating essential functions such as obstacle detection, autonomous driving assistance, and distance measurement. However, reliance on a single radar transmitter poses significant risks, including signal degradation over distance and susceptibility to environmental interference and equipment failures. Building on previous research [2], [3], [5]–[10] this paper explores how multiple follower agents, initially distributed across different locations, effectively and

stably track the signal emitted by a leader agent, enhancing the overall performance and robustness of the system.

External disturbances are prevalent in practical applications and adversely impact system performances. Disturbances arise from electromagnetic interference of devices, other vehicles, environmental factors, and coupling effects from adjacent subsystems. To strength system's resilience against disturbances, the disturbance observer-based control (DOBC) method was in [11]. A detailed introduction of the DOBC method is provided in [12], which serves as a foundation for a deeper understanding of the approach. Subsequent works [7], [13] employed Lyapunov stability theory in conjunction with the DOBC methodology to establish global stability results for nonlinear systems under mismatched conditions. Furthermore, [14] examines the effects of both matched and mismatched disturbances on control performance within the MASs framework, utilizing directed graphs. Notably, previous studies [7], [11]–[16] have overlooked unmeasurable states, which are commonly encountered in real-world systems. Consequently, this paper introduces dual observers comprising both state and disturbance observers to simultaneously tackle the challenges posed by unmeasurable states and external disturbances.

Time-triggered control schemes have been widely studied to tackle communication challenges in MASs, however, they often cause excessive resource consumption. Event-triggered control has emerged as an efficient alternative, offering advantages over periodic pulse control [17]. This approach is divided into two types based on triggering conditions: fixed threshold and relative threshold triggering. [18] provides a thorough summary of these types and proposes a switched event-triggered mechanism that combines various strengths. Furthermore, [19] extends the mechanism under state constraints and fixed time conditions. However, both [18] and [19] focus solely on nonlinear systems. Our work seeks to apply the switched event-triggered strategy within MASs to reduce each agent's controller triggering frequency, conserving resources and optimizing control performance.

Inspired by these insights, we propose an efficient control framework for the radar signal formation problem, which considers unknown states and environmental uncertainties during the control process. Our approach leverages backstepping [20], filtering [21], [22], and neural networks [23], [24]. The contributions of this paper are outlined as follows:

1) We introduce a novel dual-observer based control mechanism to address the leader-following consensus tracking control problem in radar formations, independent of the initial positions of the radar transmitting devices.

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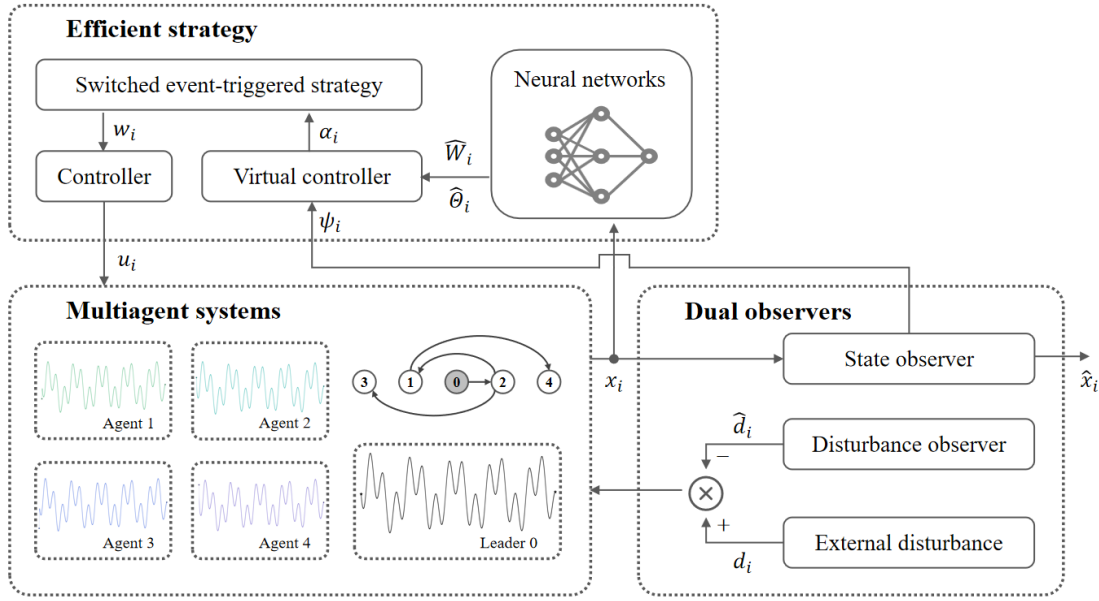


Fig. 1. Our efficient dual observer based consensus tracking control framework for MASs.

2) Given unknown states and external disturbances in applications, our approach comprises both state and disturbance observers to enhance controller feasibility. The integration backstepping and neural networks also significantly reduces the complexity of controller design.

3) We employ a switched event-triggered strategy that effectively reduces the triggering frequency, optimizes control resource utilization and extends the controller's operational lifespan, thereby achieving efficient control.

The remainder of the paper is organized as follows. Section II reviews relevant preliminaries and formulates the consensus tracking problem for investigation. Section III develops a dual observer based control approach to address the problem, followed by the stability analysis. Section IV includes a case study of radar formation to illustrate the performance of our approach. Finally, Section V concludes the work and proposes some future research directions.

II. PRELIMINARIES AND PROBLEM FORMULATION

In this section, we first present a comprehensive overview of preliminary knowledge such as graph theory, multiagent systems (MASs) model, dual observers and switched event-triggered control mechanism. Then we formulate the adaptive consensus tracking control problem in the MASs setting.

In order to describe the information transformation between the leader and N followers, according to [25], a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ are introduced with nodes $\mathcal{V} = \{v_1, \dots, v_N\}$ and edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. If $(v_i, v_j) \in \mathcal{E}$ holds, v_j can transmit information to v_i . The matrices $\mathcal{A} = [a_{i,j}]$, $\mathcal{D} = \text{diag}(d_1, \dots, d_N)$ and $\mathcal{L} \in \mathbb{R}^{N \times N}$ represent the adjacency, degree and Laplacian matrices of \mathcal{G} , with $\mathcal{L} = \mathcal{D} - \mathcal{A}$. The matrix $\mathcal{B} = \text{diag}(b_1, \dots, b_N)$ represents the information transformation between the leader and its neighbor agents. In this paper, the error term is equal to the difference between the actual value and the estimative value, $\tilde{\circ} = \circ - \hat{\circ}$.

This work investigates the consensus tracking of MASs of leader-follower settings. The ultimate objective is to ensure that the output signal of each follower aligns effectively and synchronously with the reference signal generated by the leader. The dynamics of each agent is modeled as a nonlinear equation $\dot{x} = f(x, u)$ where $x \in \mathbb{R}$ and $u \in \mathbb{R}$ are system state and control input, respectively. f is local Lipschitz with $f(0, 0) = 0$. As noted in [26], a nonlinear system possesses the bounded input bounded state attribute if, given any initial state and any continuous input being uniformly bounded, the solution exists and is consistently bounded. The specific dynamics will later be approximated by a neural network. Then we leverage the MASs model from [27] as:

$$\begin{aligned} \dot{x}_{i,k} &= x_{i,k+1} + f_{i,k}(\bar{x}_{i,k}) + d_{i,k} \\ \dot{x}_{i,n} &= u_i + f_{i,n}(\bar{x}_{i,n}) + d_{i,n} \\ y_i &= x_{i,1} \end{aligned} \quad (1)$$

Here the indexes are: $i \in \{1, \dots, N\}$, $j \in \{1, \dots, n\}$ and $k \in \{1, \dots, n-1\}$. $\bar{x}_{i,j} = [x_{i,1}, \dots, x_{i,j}]^T$ are the system states, which are not measurable. $u_i \in \mathbb{R}$ denotes the control input of the MASs. The output of the i -th follower is expressed as $y_i \in \mathbb{R}$. The function $f_{i,j}(\bar{x}_{i,j})$ represents C^1 class nonlinear smooth equation vectors. And $d_{i,j}$ represents unknown external disturbances imposed on the MASs.

Given that the MASs have unmeasurable states and are exposed to disturbances, we consider a dual observer framework for estimation, which is designed as follows:

1) State Observer: It is often intricate to implement the output-feedback control profile of MASs when system states are not measured precisely. To tackle this issue, we first construct the state observer of MASs model (1):

$$\begin{aligned} \dot{\hat{x}}_{i,k} &= \hat{x}_{i,k+1} + \hat{f}_{i,k}(\hat{x}_{i,k}) + q_{i,k}\psi_{i,1} + \hat{d}_{i,k} \\ \dot{\hat{x}}_{i,n} &= u_i + \hat{f}_{i,n}(\hat{x}_{i,n}) + q_{i,n}\psi_{i,1} + \hat{d}_{i,n} \\ \hat{y}_i &= \hat{x}_{i,1} \end{aligned} \quad (2)$$

where \hat{x} , \hat{f} , \hat{d} and \hat{y} are the estimates of x , f , d and y , respectively. $q_{i,k}$ is an observer gain, which enables the system to correct estimation errors more rapidly, thereby improving the system's response speed, while also ensuring the stability of the observer, preventing the observation errors from diverging over time. $\psi_i = [(x_{i,1} - \hat{x}_{i,1})^T, \dots, (x_{i,n} - \hat{x}_{i,n})^T]^T$ represents the error of state observation.

2) Disturbance Observer: Mismatched disturbances increase complexity of controller design and adversely impact the performance and stability of MASs. Therefore, it is essential to enhance disturbance rejection capabilities to achieve high-precision control in the presence of such disturbances. To address the dynamical disturbances present in system (1), we construct a disturbance observer as follows:

$$\hat{d}_{i,l} = \hat{\tau}_{i,l} + \kappa_{i,l} \hat{x}_{i,l} \quad (3)$$

where for $l \in \{1, \dots, n\}$, $\tau_{i,1}$ represent an auxiliary variable and $\kappa_{i,l}$ is a designated positive constant.

To reduce the frequency of controller activation, we introduce a switched event-triggered strategy as [18]. This strategy balances the controller performance by incorporating both fixed and relative thresholds. It allows extended update intervals when signal magnitudes are large while enhancing precision as signals approach zero. Thus, this approach optimizes the system performance across a broad spectrum of signal magnitudes. The adaptive control law w_i is detailed in Section III and the triggering event is defined as:

$$u_i(t) = w_i(t_k) \quad (4)$$

$$t_{k+1} = \begin{cases} \inf \{t \in R | |\mu_i| \geq \varphi\}, & |u_i| < G \\ \inf \{t \in R | |\mu_i| \geq \delta |u_i| + \varphi\}, & |u_i| \geq G \end{cases}$$

where t_k denotes the triggering time when the controller is updated, with $k \in \mathbb{Z}$. $\mu_i = u_i - w_i$ denotes the measurement error of controllers. Here, φ and δ are positive design constants, with the constraint $0 < \delta < 1$, and G represents the designed switching gate. Additionally, $\alpha_{i,n}$ denotes the virtual controller, while $z_{i,n}$ represents the tracking error, both of which will be further elaborated upon in Section III.

We consider the relationship between the length of each transmitter and the spatial arrangement of adjacent transmitters, which is crucial for adaptive consensus tracking control.

Problem 1 (Adaptive consensus tracking of radar transmitters). *Given a team of radar transmitters modeled as MASs consisting of N followers and one leader where their mutual communication is represented by a directed graph, then subject to unknown external perturbations d , each follower is expected to accurately and efficiently track the reference signal y_r generated by the leader to achieve consensus.*

Furthermore, the following results from the preliminaries of [27] are leveraged to facilitate the controller synthesis.

Lemma 1. *Inequality $0 \leq |\chi| - \chi \tanh(\frac{\chi}{\chi_0}) \leq 0.2785\chi_0$ holds for any specified parameter $\chi \in R$ and $\chi_0 > 0$.*

Assumption 1. *In this MASs, the leader's desired reference signal y_r is measurable, smooth and bounded, and the unmeasured external disturbances are all bounded.*

III. DUAL OBSERVER BASED CONSENSUS CONTROL

In this section, we develop an efficient dual observer based control approach to achieve consensus tracking for radar transmitters in the presence of model and environment uncertainties. First, a class of neural networks (NNs) is designed to approximate the unknown nonlinear system dynamics. Next, we combine the output of the NNs and the estimation from the state observer to design a virtual controller following the backstepping method. Then a switched event-triggered adaptive control strategy is proposed to solve the leader-following consensus tracking problem, reducing triggering frequency and achieving high efficiency. Finally, we prove the convergence and stability of our approach. The general framework of our approach is illustrated in Fig. 1.

Given the multiagent systems (MASs) model (Equation (1)) in Section II, we define the graph-based errors $z_{i,k}$ and the boundary layer errors $e_{i,k}$ as: for $k \in \{1, \dots, n-1\}$,

$$z_{i,1} = \sum_{j=1}^N a_{i,j}(y_i - y_j) + b_i(y_i - y_r) \quad (5)$$

$$z_{i,k} = x_{i,k} - \bar{\alpha}_{i,k}$$

$$e_{i,k} = \bar{\alpha}_{i,k} - \alpha_{i,k}$$

The terms $\alpha_{i,k}$ and $\bar{\alpha}_{i,k}$ denote the virtual control and its filtered counterpart, respectively. In the context of adaptive control, we employ radial basis function neural networks (RBFNNs) to approximate nonlinear functions and to manage uncertainties inherent in MASs. This approach enables the system to adapt effectively to changes while maintaining optimal performance under uncertain conditions. Using NNs to approximate the unknown nonlinear dynamics in MASs (1), we consider $\sigma_{i,l}(t)$ as the bounded approximation error. The unknown smooth nonlinear function is then expressed as $f_{i,l}(\bar{x}_{i,l}) = W_{i,l}^{*T} E_{i,l}(\hat{x}_{i,l}) + \sigma_{i,l}(t)$. We also define $\Omega_{i,l}$, $\hat{\Omega}_{i,l}$ and $\tilde{\Omega}$ as compact sets corresponding to $\bar{x}_{i,l}$, $\hat{x}_{i,l}$ and $\hat{E}_{i,l}$, respectively. In addition, we let $\Theta_i^* = \max\{\|W_{i,l}^*\|\}$. Then the optimal weight $W_{i,l}^*$ for $1 \leq l \leq n$ is designed as:

$$W_{i,l}^* = \arg \min_{\hat{W}_{i,l} \in \tilde{\Omega}} \sup_{\substack{\bar{x}_{i,l} \in \Omega_{i,l} \\ \hat{x}_{i,l} \in \hat{\Omega}_{i,l}}} \left| \hat{f}_{i,l}(\hat{x}_{i,l}) - f(\bar{x}_{i,l}) \right| \quad (6)$$

The aforementioned error terms are established to facilitate subsequent stability analysis. If the derivative of the Lyapunov function, composed of these error terms, is zero at the equilibrium point and positive elsewhere, with a negative time derivative, it indicates that the errors are controlled within a specific range, thereby ensuring the asymptotic stability of the system. To formalize this, we define a Lyapunov function with a designed positive constant $\eta_{i,k}$ as:

$$V_{i,k} = \frac{z_{i,k}^2}{2} + \frac{\tilde{\tau}_{i,k}^2}{2} + \frac{1}{2\eta_{i,k}} \tilde{W}_{i,k}^T \tilde{W}_{i,k} \quad (7)$$

Following the framework in [19], we enhance the robustness and adaptability of the system by employing *backstepping method* to recursively design adaptive controllers. This approach allows us to decompose the system into multiple subsystems, each governed by a *virtual controller*.

The design process initiates with the simplest subsystem and progressively advances to address more complex controllers. We introduce an auxiliary function processed by NNs to produce an optimal weight $E_{i,k}$, analogous to $W_{i,k}$. Furthermore, $\bar{\alpha}_{i,k}$ is derived by applying a first-order low-pass filter to the virtual controller $\alpha_{i,k}$. At each step, we incorporate the designed constants $r_{i,k}$, $h_{i,k}$, $m_{i,k}$, $c_{i,k}$ and the NN input as $T_{i,k} = [y_r, \hat{\Theta}_i, \hat{x}_{i,k}^T, \hat{x}_{j,k}^T, \hat{\omega}_{i,k}^T, \hat{\omega}_{j,k}^T]^T$. Then the virtual controller and adaptive parameter are as:

$$\alpha_{i,k+1} = r_{i,k} z_{i,k} - \frac{z_{i,k}}{2} + \frac{\alpha_{i,k} - \bar{\alpha}_{i,k}}{m_{i,k}} - q_{i,k} \psi_{i,1} - \frac{\hat{\Theta}_i}{2c_{i,k}^2} z_{i,k} E_{i,k}^T(T_{i,k}) E_{i,k}(T_{i,k}) \quad (8)$$

$$\dot{\hat{W}}_{i,k} = -h_{i,k} \hat{W}_{i,k} - \eta_{i,k} \tilde{\tau}_{i,k} \kappa_{i,k} E_{i,k}(\hat{x}_{i,k})$$

Our event-triggered control law integrates relative-threshold and fixed-threshold strategies, where a *switching gate* determines which strategy to deploy. When the values of control signal are large, the relative-threshold strategy is utilized to increase the triggering interval, mitigating the excessive influence of signal spikes on the controller and enhancing its robustness against disturbances. Conversely, when the controller signal values fall below the gate threshold, the fixed-threshold strategy takes over to set a triggering interval which steadily reduces the number of controller activations and conserves control resources. The overall controller synthesis process is summarized in Algorithm 1. Next, we propose adaptive event-triggered controllers following these strategies with constants $\bar{\varphi}$ ($\bar{\varphi} > \varphi/(1-\delta)$) and Γ :

$$w_i(t) = \alpha_{i,k+1} - \bar{\varphi} \tanh\left(\frac{z_{i,k} \bar{\varphi}}{\Gamma}\right) \quad (10a)$$

$$w_i(t) = -(1+\delta)(\alpha_{i,k+1} \tanh\left(\frac{z_{i,k} \alpha_{i,k+1}}{\Gamma}\right) + \bar{\varphi} \tanh\left(\frac{z_{i,k} \bar{\varphi}}{\Gamma}\right)) \quad (10b)$$

Finally, we apply Lyapunov stability theory to demonstrate that all signals in the controlled system remain bounded. Furthermore, we also validate the performance of the event-triggered control and confirm the absence of Zeno behavior.

Theorem 1. Consider the MASs (1), the dual observers are designed by (2) and (3), the virtual controller and adaptive parameter are constructed as (8). Provided that the initial conditions are confined to a compact set, then (i) all signals of the closed-loop system remain uniformly bounded; (ii) the consensus tracking errors between follower outputs and the leader's trajectory signal are reduced within a designated range; (iii) Zeno behaviors are effectively prevented.

Proof: We first prove (i) and (ii). In terms of (7), we define the overall Lyapunov function for the MASs as

$$V = \sum_{i=1}^N \sum_{k=1}^n V_{i,k} + \sum_{i=1}^N \sum_{k=1}^{n-1} \frac{e_{i,k+1}^2}{2} \quad (11)$$

Taking the derivative of V , with negative unmeasured parameters $r_{i,k}^*$, $\kappa_{i,k}^*$, positive parameters $m_{i,k}^*$, $h_{i,k}$, φ_i , λ_i , $\iota_{i,k}$, o_i , Ξ and applying Young's inequality gives:

$$\begin{aligned} \dot{V} &\leq -\beta V + \gamma \\ \beta &= \min \{-2r_{i,k}^*, -4\kappa_{i,k}^*, 2m_{i,k}^*, h_{i,k}, \varphi_i, \lambda_i\} \\ \gamma &= \sum_{i=1}^N \left[\sum_{k=1}^n (\iota_{i,k} + \frac{h_{i,k}}{2\eta_{i,k}} W_{i,k}^{*T} W_{i,k}^*) + \sum_{k=1}^{n-1} \frac{\Xi}{2} + \frac{\lambda_i}{2o_i} \Theta_i^{*2} \right] \end{aligned} \quad (12)$$

When set $\beta > \gamma/\Omega$, we get $\dot{V} < 0$ on $V = \Omega$. Further, if at time $t = 0$ the condition $V \leq \Omega$ holds, it follows that $V \leq \Omega$ for entire $t > 0$. This demonstrates that the error signals are uniformly bounded. It is straightforward to derive the following equation $\frac{1}{2} \|\Upsilon_1\|^2 \leq V(t) \leq e^{-\beta t} V(0) + \frac{\gamma}{\beta} (1 - e^{-\beta t})$, with $\Upsilon = [\Upsilon_1^T, \Upsilon_2^T, \dots, \Upsilon_N^T]^T$.

Then we show (iii). Based on the above stability analysis, it is imperative that there exists a positive parameter h such that $|\dot{w}_i(t)| \leq h$ following results in [27]. Furthermore, for the time interval $t \in [t_k, t_{k+1})$, the lower bound for the inter-execution time is given by $t^* \geq \max\{\varphi, \delta|u_i| + \varphi\}/h$. Consequently, the Zeno behaviors are avoided.

Algorithm 1: Switched event-triggered control

Input: $N, n, T, t, \varphi, \delta, G$

Output: u_i

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1 for  $i$  in  $[N]$  do
2   # initial agent;
3   for  $k$  in  $[n-1]$  do
4     Calculate the tracking error  $z_{i,k}$ , the virtual
       controller and the adaptive parameter  $E_{i,k}$ 
       by (5) and (8);
5   for  $t$  in  $[T]$  do
6     if  $|u_i| < G$  and  $|\mu_i| \geq \varphi$  then
7       Calculate  $w_i(t)$  by Eq.(10a);
8       let  $u_i(t) = w_i(t)$ ;
9     if  $|u_i| \geq G$  &  $|\mu_i| \geq \delta|u_i| + \varphi$  then
10      Calculate  $w_i(t)$  by Eq.(10b);
11      let  $u_i(t) = w_i(t)$ ;
12    else
13      let  $u_i(t) = u_i(t)$ ;

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IV. CASE STUDY

We present a case study to evaluate the performance of our proposed control scheme on a radar formation problem. In this scenario, each radar transmitter is modeled as an agent of a multiagent system, including four followers and one leader. The communication topology is depicted in Fig. 1. Following the design outlined in [28], the trajectory signal of the leader is $y_r = -0.5 \sin(3t + 0.5\sqrt{t}) \cos(6t + 0.3\sqrt{t})$.

Practically, multiple radar transmitters are employed and it is essential to maintain a specific distance between each transmitter. So we design the initial positions of the followers to maintain these distances as: $x_1(0) = -0.2$, $x_2(0) = 0.2$, $x_3(0) = -0.4$ and $x_4(0) = 0.4$. To validate the reliability of the state observer, we also ensure that the initial positions of the observers differ from those of the agents, specifically: $\hat{x}_1(0) = 0.3$, $\hat{x}_2(0) = -0.3$, $\hat{x}_3(0) = 0.5$ and $\hat{x}_4(0) = 0.1$.

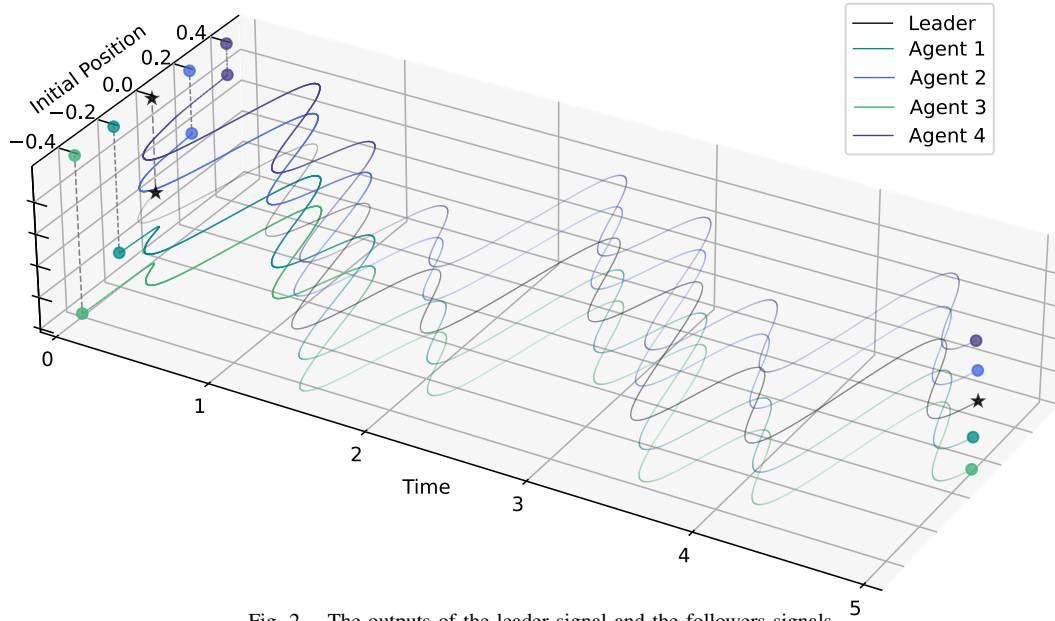


Fig. 2. The outputs of the leader signal and the followers signals.

The communication topology is shown in Fig. 1. The connection matrix linking the leader to the followers is represented as $\mathcal{B} = \text{diag}\{0, 1, 0, 0\}$. And the adjacency matrix \mathcal{A} and the Laplacian matrix \mathcal{L} are presented as

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \mathcal{L} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

In addition, the system dynamics and disturbance are modeled as $f_{i,1}(\bar{x}_{i,1}) = 0.8x_{i,1}e^{-1.4x_{i,2}^2}$, $f_{i,2}(\bar{x}_{i,2}) = -0.5x_{i,1}^2 \cos(x_{i,2})$, $d_{i,1} = 0.8x_{i,1} \sin(x_{i,2}) \cos^2(t)$ and $d_{i,2} = 0.2x_{i,2} \cos(x_{i,1}) \cos^2(t)$, $i = 1, \dots, 4$. Based on the parameter selection guidelines outlined in the stability analysis, the quantitative values of the designed constants are determined as follows, for observer constants $q_i = 333$, $\kappa_i = 6$, for triggered strategy constants $\varphi = 2.5$, $\delta = 0.25$, $G = 6$, $\bar{\varphi} = 4$, for other constants $\eta_{i,1} = \eta_{i,2} = 0.01$, $r_{i,1} = r_{i,2} = -80$, $h_{i,1} = h_{i,2} = 5$, $c_{i,1} = c_{i,2} = 120$ and $m_i = 0.003$. The following are the illustrative results and the corresponding analysis of the table and figures.

The following table includes the ten-time average trigger frequency for each agent under the switched event-triggered strategy. The control frequency reduction (CFR) rate indicates a significant decrease in the number of triggers, while maintaining satisfactory tracking performance. On average, control efficiency is improved by 93.48%, showing effective reduction of control resource utilization under our strategy.

Agents	Fixed	Relative	Switched	Total	CFR rate
Agent 1	26	624	650	10000	93.50%
Agent 2	28	599	627	10000	93.73%
Agent 3	25	632	657	10000	93.43%
Agent 4	35	641	676	10000	93.24%

The simulation results are depicted in Figs. 2 to 4. Fig. 2 illustrates the tracking performance of the leader radar's trajectory signal in relation to the four radar transmitter agents,

each starting from different initial positions. The results demonstrate that the proposed method ensures satisfactory tracking performance. To further validate the effectiveness of the state observers, Fig. 3 presents the state observation results for each agent, confirming that the adaptive state observers can effectively detect unmeasurable system states. Additionally, Fig. 4 shows the observation error of external unknown disturbances, which remains bounded. Based on these simulation results, we conclude that the proposed method is effective for multiagent systems (MASs) subject to unmeasured system states and external disturbances.

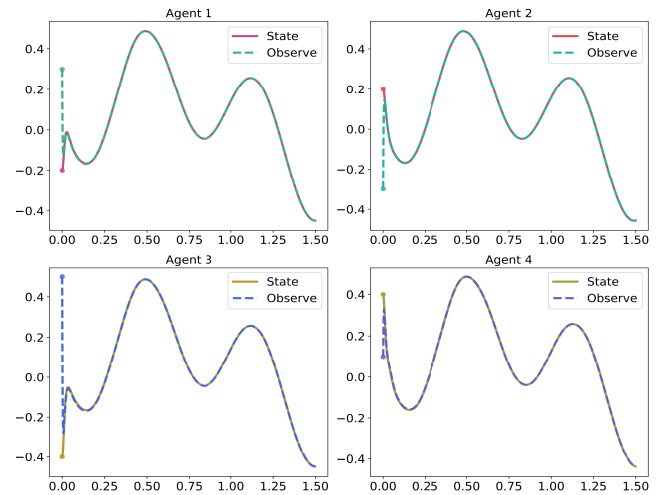


Fig. 3. The outputs of state observers

Remark 1. Simulation results validate the effectiveness of our proposed adaptive dual observer based leader-following consensus control scheme for nonlinear MASs, achieving satisfactory tracking performance despite the presence of unknown disturbances. Furthermore, parameters of simulation significantly influence the results; tuning observer gains, triggering thresholds, and control constants affect the

activation frequency and system stability. Properly calibrated parameters enhance resource utilization, disturbance rejection and the overall performance of the closed loop system.

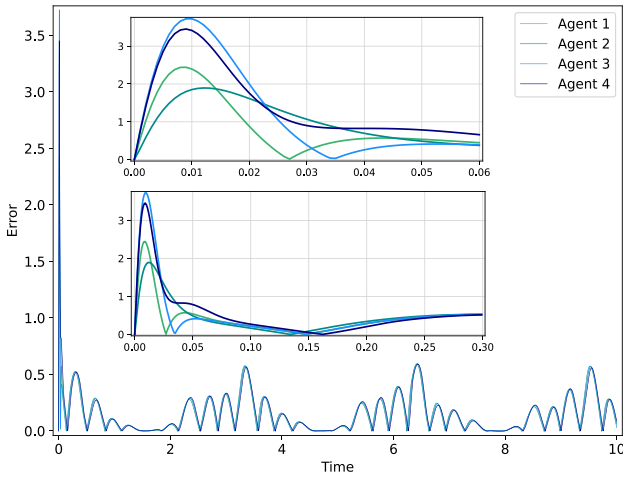


Fig. 4. The errors of disturbance observers

V. CONCLUSION

Motivated by the challenges associated with signal tracking of radar systems under external disturbances, we propose an adaptive observer based leader-following consensus control scheme using a switched event-triggered strategy for nonlinear multiagent systems. The dual observer accurately estimates the states of each agent and the external disturbances. Next, we introduce a special neural network to approximate the dynamics of the system. Based on that, an adaptive control strategy is developed, which switches between two strategies under the triggering condition. The designed controllers ensure that the system achieves both bounded and asymptotic consensus tracking performance. A case study of radar transmitter formation control is provided to demonstrate the practical performance of our dual-observer approach. For future research, we plan to extend our framework to more complicated settings with communication delays, state constraints, and fixed-time convergence.

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