Study on Vehicle Stability Control by Using Model Predictive Controller and Tire-road Force Robust Optimal Allocation

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Study on Vehicle Stability Control by Using Model Predictive Controller and Tire-Road Force Robust Optimal Allocation

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ABSTRACT
The vehicle chassis integrated control system can improve the stability of vehicles under extreme conditions using tire force allocation algorithm, in which, the nonlinearity and uncertainty of tire-road contact condition need to be taken into consideration. Thus, An MPC (Model Predictive Control) controller is designed to obtain the additional steering angle and the additional yaw moment. By using a robust optimal allocation algorithm, the additional yaw moment is allocated to the slip ratios of four wheels. An SMC (Sliding-Mode Control) controller is designed to maintain the desired slip ratio of each wheel. Finally, the control performance is verified in MATLAB-CarSim co-simulation environment with open-loop maneuvers.


INTRODUCTION
The past 30 years have witnessed the development of vehicle chassis control systems [1]; from single wheel control system (Anti-lock Braking Control-ABC, Traction Control System-TCS) to vehicle system dynamics control (Electronic Stability Control-ESC [2]) and chassis integrated control systems [3, 4]. Nowadays, the vehicle chassis controllers can be classified into three categories according to the tire force direction: lateral tire force control (Active Front Steering - AFS, Four Wheel Steer Control - 4WS), longitudinal tire force control (ABS, TCS, Vehicle Stability Control - VSC, Vector Torque Control-VTC) and vertical tire force control (Continuous Damping Control - CDC, Anti Roll Control - ARC, Active Rear Spoiler Control).

Braking/driving and steering are two important control systems on longitudinal and lateral directions that have attracted many researchers. The coordination between ESC and EPS (Electric Power Steering) are studied by using the limitation information of tires calculated from EPS as the reference of ESC controller to avoid saturation of tire forces [5]. This kind of integrated control is based on the information interaction of different systems. Also, another kind of controller was proposed by many researchers, of which the 4WS control is of higher priority. When the control objective is beyond the ability of 4WS, the ESC controller or ESC& 4WS integrated controller which is combined with the active front steering and braking force allocation algorithm [6, 7] works. Finally, the integrated controllers considering the integration of different control inputs are also discussed in many papers, which are based on multi-input and multi-output vehicle model with the non-linear tire model taken into consideration [8, 9]. This paper will focus on an integrated chassis controller aiming at improving the vehicle handling stability.

By using model predictive control, the vehicle will be able to follow the driver's steering intent better so that the handling stability can be improved [10, 11]. For the model based control algorithm, an accurate model is highly valuable because the model error will bring down the performance of the vehicle especially when the tires enter non-linear region. Considering the computation burden, a compromise between the accuracy and the model complexity is necessary [12]. In this paper, the LTV-MPC (Linearized Time Varying MPC) controller adopts a single track linear model for the computational consideration. The outputs of the LTV-MPC controller are additional steering angle and additional yaw moment.

In order to allocate the additional yaw moment to the slip-ratios of each wheel, a robust optimal allocation algorithm is designed to ensure the robustness of the vehicle stability controller proposed in this paper. Selected single wheel braking logic and 2-4 braking channel with quadratic programming (QP) or nonlinear programming are implemented by many researchers [13, 14, 15]. However, the effect of the optimal braking force allocation largely relies on the force capacity of each wheel and vehicle parameters, which can hardly be obtained accurately. Hence, a robust optimal algorithm is proposed to deal with the uncertainty of the vehicle parameters and the mismatch of the tire model in this paper. Considering the saturation phenomena of them and the complicated relationship between the tire longitudinal force and lateral force, the Burkhardt tire model [16] is adopted and the slip-ratios of four wheels are allocated.

Finally, the allocated slip-ratios need to be tracked by each wheel quickly and accurately. The most commonly used slip-ratio tracking algorithm is logic threshold control [17], which can gain good control performance without depending on dynamics models. However, it
lacks theoretical base and requires lots of tuning work. The PID logic \[ \text{[18]} \] is also widely used in vehicle dynamics controller. However, the parameters of PID controller highly rely on the tire-road contact condition. Thus it cannot guarantee consistent control performance without knowing the road conditions. In this paper, considering the non-linearity of the tire and uncertainty of tire-road contact situation, an SMC controller is used, which is insensitive to model parameters and responses quickly if finely tuned.

The entire structure of the proposed hierarchical controller is shown in Figure 1:

![Figure 1. Configuration of the control system](image)

The following paper is structured as follows: First, the design of the MPC controller will be introduced. Then, the optimal robust allocation algorithm will be described. Next, the sliding-mode algorithm to track the slip-ratio of the wheel will be proposed. Finally, we will present and analyze the simulation results and a conclusion will be made at the end of this paper.

**THE MPC CONTROLLER**

**Vehicle Model and Tire Characteristics**

In this section, the vehicle models are depicted. A linear bicycle model is used to generate the reference yaw velocity and the vehicle side slip angle. And a time-varying linearized model is used as the predictive model of the MPC controller. Both of them are based on the single track assumption, as shown in Figure 2:

![Figure 2. Single track bicycle model](image)

where, $v$ represents the velocity at the center of gravity; $v_x$ and $v_y$ are the longitudinal and lateral velocity, respectively; $\gamma$ is the vehicle yaw rate and $M_z$ is the additional yaw moment applied on the vehicle; $\beta$ is the vehicle side slip angle; $a$, $b$ represent the distances of the front axle and the rear axle from the center of gravity; $F_{yf}$ and $F_{yr}$ are the lateral forces of the front and the rear axles; $a_f$ and $a_r$ are the tire side slip angles of the front and the rear axles; $\delta_f$ is the steering angle of the front wheel.

**Linear Bicycle Model and Reference Yaw Rate & Sideslip Angle**

As shown in Figure 2, supposing the vehicle speed $v_x$ is constant, then the vehicle dynamics equations are as follows:

\[
\dot{\beta} = \frac{F_{yf} + F_{yr}}{mv_x} - \gamma
\]

\[
\ddot{\gamma} = \frac{a F_{yf} - b F_{yr}}{I_{mx}}
\]

While, the lateral force of the front and the rear axles are presented as follows:

\[
F_{yf} = -C_{a\beta} a_f F_{yf}, F_{yr} = -C_{a\beta} a_r F_{yr}
\]

where, $C_{a\beta}$ and $C_{a\beta}$ are the cornering stiffness of the tires obtained by linearizing the nonlinear curve of tire force from the origin. The tire slip angles of the front and the rear axles are:

\[
a_f = \beta + \frac{av_x}{v_x} - \delta_f, a_r = \beta - \frac{bv_x}{v_x}
\]

Defining $x = [\beta, \gamma]^T$ and substituting Eq. (3) and Eq. (4) into Eq. (1) and Eq. (2), the linear bicycle model can be described in a linear state-space form:

\[
\dot{x} = Ax + Bu
\]

where,

\[
A = \begin{bmatrix}
-(\frac{a_f a_r + \gamma}{v_x}) & \frac{a_r}{v_x} \\
-(\frac{a_f a_r - \gamma}{v_x}) & \frac{a_r}{v_x}
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
\frac{C_{a\beta}}{m v_x} \\
\frac{C_{a\beta}}{I_{mx}}
\end{bmatrix}
\]

Hence, the reference output can be obtained as:

\[
y_{N0} = v_x \delta_f / [(a + b) \left(1 + \left(\frac{v_x}{v_{ch}}\right)^2\right)]
\]

where, $v_{ch} = \frac{C_{a\beta} (a + b)^2}{m (a \beta - C_{a\beta} a)}$. The maximal tire-road friction force will limit the reference value,

\[
y_{\text{ref max}} = \frac{\mu g}{v_x} |\text{sign}(\delta)|
\]
where, $\mu$ is the coefficient of friction, $g$ is the acceleration of gravity. Hence, the reference vehicle yaw rate is given by

$$\gamma_{ref} = \min(\gamma_{N0}, [\mu g/v_0])\text{sign}(\delta)$$

(9)

The vehicle slip angle is also limited by a function of vehicle speed, which, if exceeded, the vehicle will lose control [19]:

$$\beta_{max} = 10^\circ - 7^\circ \frac{v_x^2}{v_{ch}^2}$$

(10)

So, the reference vehicle side slip angle is given by:

$$\beta_{ref} = \{\beta, |\beta| < |\beta_{max}| \pm \beta_{max}, |\beta| \geq |\beta_{max}|$$

(11)

Then the reference yaw rate and sideslip angle are as follows:

$$\gamma_{ref} = [\beta_{ref}, \gamma_{ref}]$$

(12)

**Linearized Time Varying Bicycle Model**

As the cornering stiffness is constant in the above bicycle model, it can only predict the dynamic response in linear area. In order to keep the predictive precision in both linear and non-linear regions, a linearized time varying bicycle model is applied.

Aiming at reflecting the accurate tire characteristics, the tire cornering stiffness is defined as the slope at the working point, which is called the instantaneous cornering stiffness $C_{in}$, as shown in Figure 3. For the front and rear axles:

$$F_{yi} = F_{yi,0} - C_{in,0}(f_i - a_{i,0})$$

where, $F_{yi,0}$ and $a_{i,0}$ represent the lateral force and the tire slip angle at the working point, respectively. $C_{in,0}$ are the instantaneous cornering stiffness of the front and the rear axles, which can be calculated online from the tire model. In this paper, the Burckhardt tire model is used, which will be discussed in following sections.

Then we substitute Eq. (13) into Eq. (1) and Eq. (2) so that a linearized bicycle model can be obtained with the tire model linearized at the working point:

$$\dot{\beta} = \frac{\beta_{f0} - \beta_{f0}(a_1 - a_{i,0}) + \beta_{r0} - \beta_{r0}(a_r - a_{r,0})}{mv_x} - \gamma$$

$$\dot{\gamma} = \frac{a(\beta_{f0} - \beta_{f0}(a_1 - a_{i,0}) - b(\beta_{r0} - \beta_{r0}(a_r - a_{r,0}) + M_z)}{I_{zz}}$$

(14)

(15)

When the active steering angle $\delta_{f, add}$ is taken into consideration, the front wheel steering angle can be written as:

$$\delta_f = \delta_{f, driver} + \delta_{f, add}$$

(16)

where, $\delta_{f, drive}$ is the steering angle of front wheel exerted by the driver. Denoting $x = [\beta, \gamma]^T$ and $u = [\delta_{f, add}, M_z]^T$, an LTV state space model can be obtained from the equations above:

$$\dot{x} = A_{c,t}x + B_{c,t}u + d_{c,t}$$

(17)

where,

$$A_{c,t} = \begin{bmatrix} -\dot{C}_{in,0} + \dot{\gamma}_{in,0} & -a\dot{C}_{in,0} + b\gamma_{in,0} - \frac{1}{I_{zz}} \end{bmatrix}, B_{c,t} = \begin{bmatrix} \dot{C}_{in,0} & 0 \\ a\dot{C}_{in,0} & \frac{1}{I_{zz}} \end{bmatrix}$$

$$d_{c,t} = \begin{bmatrix} \dot{\beta}_{f0} + \dot{\gamma}_{in,0}(a_{f,0} + \delta_{f, driver}) + \dot{\beta}_{r0} + \dot{\gamma}_{in,0}(a_{r,0} + \delta_{r, driver}) \\ \frac{a(\dot{\beta}_{f0} + \dot{\gamma}_{in,0}(a_{f,0} + \delta_{f, driver}) - b(\dot{\beta}_{r0} + \dot{\gamma}_{in,0}(a_{r,0} + \delta_{r, driver}) + M_z)}{I_{zz}} \end{bmatrix}$$

(18)

d_{c,t} is considered to be constant in every sampling step. The tire slip angles of each axle are obtained through Eq. (4). In this paper, we assume that the steering angle $\delta_{f, drive}$ the slip angle of the vehicle $\beta$ and the yaw rate of the vehicle $\gamma$ can be measured directly.

**Design of the MPC Algorithm**

The nominal vehicle slip angle and the yaw rate are generated from Eq. (5). The linearized vehicle model described in Eq. (17) is used as the predictive model with sampling time $T_s$. In the predictive model, the cornering stiffness of the tire is updated each time at working points and the steering angle of the front wheel exerted by the driver is assumed to be constant in each step. The outputs of the MPC controller are additional front wheel steering angle $\delta_{f, add}$, and additional yaw moment $M_z$, of which, the former is applied to the vehicle directly and the latter is used as the input of the robust optimal allocation algorithm. The MPC control algorithm is described as follows:

![Figure 3. The relationship between tire lateral force and tire slip angle](image-url)
Vehicle Model with Non-linear Tire Model

First, a tire model is selected. Commonly used tire models include Dugoff Model [20], Brush Model [21], Burckhardt Model [22], Pacejka Magic Formula Model [23], etc. To reduce the complexity of derivation process, the Burckhardt Model is adapted here, which can be expressed as:

\[
\mu_{\text{Res}}(S_{\text{Res}}) = \left( c_4[1 - \exp(-c_2S_{\text{Res}})] - c_3S_{\text{Res}} \right) e^{-c_4v}
\]

where, \( \mu_{\text{Res}} \) is the comprehensive coefficient of friction. \( c_1, c_2, c_3, c_4 \) are coefficients that are decided by tire-road conditions and can be fitted based on experimental data. Among them, \( e^{-c_4v} \) reflects the influence of vehicle speed and, usually, \( c_4 = 0.02 \sim 0.04 \text{ s/m} \), which will be ignored in this paper. \( S_{\text{Res}} \) is the comprehensive tire slip ratio and can be defined as follows:

\[
S_{\text{Res}} = \sqrt{S_L^2 + S_S^2}
\]

\( S_L \) and \( S_S \) are longitudinal ratio and lateral slip ratio respectively and in the braking condition, they can be expressed as follows:

\[
S_L = \frac{(v_R \cos \alpha - v_w)/v_w}{1 + S_S \tan \alpha}
\]

where, \( v_R \) is the linear velocity of wheel rotation, which can be measured directly. \( v_w \) is the linear velocity at the wheel center. \( \alpha \) is the tire slip angle. Both \( \alpha \) and \( v_w \) can be obtained easily from the knowledge of the vehicle longitudinal speed \( v_x \), the lateral speed \( v_y \), the steering angle of the front wheel \( \delta \), and the vehicle yaw rate \( \gamma \). For simplification, the steering angle of left and right front wheels are assumed to be the same. Thus, the longitudinal force and the lateral force of tires in the tire coordination \( Y_w-O_w-X_w \) can be obtained, as shown in Figure 4:

\[
f_L = \frac{S_L \mu_{\text{Res}} F_z}{S_{\text{Res}}}
\]

\[
f_S = k_S \frac{S_S \mu_{\text{Res}} F_z}{S_{\text{Res}}}
\]

\( k_S \) is a coefficient that indicates the difference between the longitudinal and lateral tire forces. Usually, \( k_S = 0.9 \sim 0.95 \) [18]. In this paper, we choose \( k_S = 0.95 \). \( F_z \) is the vertical tire force, which is assumed to be measurable in this paper. A 225/65 R17 tire is chosen and tire experiments are conducted in CarSim on both low-\( \mu \) (\( \mu = 0.4 \)) and high-\( \mu \) (\( \mu = 0.8 \)) roads. The data is fitted using nonlinear regression tool in MATLAB.
Based on the obtained tire model, the vehicle dynamics can be described by following equations, as shown in Figure 4:

\[
M_z = a(F_{yf} + F_{yr}) - b(F_{yf} + F_{yr}) + \cdots \\
+ w_i(-F_{xf} + F_{xr} - F_{xr} + F_{xrr})
\]

\[ (32) \]

where, \( w_i = df = dr \) is the wheel base of both front and rear axles. \( F_{yi} \) (\( i = f, r, j = l, r \)) are longitudinal and lateral tire forces in the vehicle body coordination and they can be expressed as follows:

\[
F_{xfj} = f_{xfj} \cos(\delta_{f,driver}) - f_{yfj} \sin(\delta_{f,driver})
\]

\[ (33) \]

\[
F_{yfj} = f_{xfj} \sin(\delta_{f,driver}) + f_{yfj} \cos(\delta_{f,driver})
\]

\[ (34) \]

\[
F_{xjr} = f_{xjr}
\]

\[ (35) \]

\[
F_{yrj} = f_{yrj}
\]

\[ (36) \]

where, \( F_{yi} \) (\( i = f, r, j = l, r \)) are obtained from Eq.(30) and (31).

Now, the yaw moment can be presented as a non-linear function of slip ratios of four wheels according to Eq. (26), Eq. (27), Eq. (28), Eq. (29), Eq. (30), Eq. (31), Eq. (32), Eq. (33), Eq. (34), Eq. (35), Eq. (36):

\[
M_z = g(S_{Ll}, \alpha_{ij}, v_x, v_y, \gamma, F_z, \delta_{f,driver}, \delta_{f,add})
\]

\[ (37) \]

where: \( i = f, r; j = l, r \)

\[ \text{where, } a_p, v_x, v_y, \gamma, F_z \text{ and } \delta_{f,driver} \text{ are regarded as constants in each step and updated according to the current working condition. Then the Jacobi Matrix of Eq. (37) can be obtained and } M_z \text{ can be expressed as follows:} \\
M_z = Au
\]

\[ (38) \]

where, \( u=[S_{Ll}, S_{Ll}], M_z - M_y, M_z \) and \( A \) is the efficiency matrix for robust allocation.

**Design of the Robust Optimal Allocation Algorithm**

Because of the uncertainty of vehicle and tire-road contact parameters and the linearization process of getting the efficiency matrix \( A \), we assume that the efficiency matrix is of 10% uncertainty. At the same time, the slip ratios of each wheel and their changing rates need to be limited. In order to reserve more tire force potential of each wheel, the quadratic sum of slip ratios is used as one of the goal functions so that the tire slip ratios can be allocated as evenly as possible. Also, the difference between the nominal yaw moment and allocated yaw moment should be reduced so that the vehicle can follow the nominal yaw rate with higher accuracy. Then, the robust optimal allocation problem can be generalized as follows:

\[
\text{Min } U^T H U
\]

\[ (39) \]

s.t. \( M_z = Au \)

\[ (40) \]

where, \( A \in \{A: A = [a_y], a_y = \hat{a}_y(1+i), \forall t \in [-\epsilon, \epsilon] \} \)

\[
S_{min} < |S_{Ll}| < S_{max}
\]

\[ (41) \]

\[
dS_{min} < dS_{Ll} < dS_{max}
\]

\[ (42) \]

where, \( U = [S_{Ll}, S_{Ll}], S_{Ll}, S_{Ll}, S_{Ll}, M_y - M_y, M_y ] \), \( M_y \) is the nominal yaw moment obtained from the MPC controller. \( H \) is a 5×5 diagonal matrix representing the weighting coefficients of each component in \( U \). \( S_{min} \) and \( S_{max} \) are the minimal and maximal allowing slip ratios. Considering that the slip ratio of a wheel is always positive even without braking, \( S_{max} \) is set to be a certain small positive value to guarantee the control effect. Moreover, \( S_{max} \) cannot exceed 1 in braking condition. \( dS_{min} \) and \( dS_{max} \) are the minimal and maximal allowing changing rates of slip ratios, which are decided by the tracking ability of the SMC controller. \( \epsilon \) represents the uncertainty of the efficient matrix. \( \hat{a}_y \) are the nominal values in \( A \) and are assumed to be independent of each other. In this paper, we set \( \epsilon = 0.1 \). The problem can be converted into a Quadratic Programming problem and solved by using YALMIP toolbox [24]. The outputs of the allocation algorithm are slip ratios of four wheels and an SMC controller will be designed to track them in next section.
BRAKING FORCE SMC CONTROLLER

Wheel Dynamic Model

Before designing the SMC controller, a dynamic model of the wheel in braking condition needs to be built, as shown in Figure 5:

\[ I_\omega \dot{\omega} = r F_x - T_b + T_d \]  

(43)

where, \( I_\omega \) is the rotational inertia of the wheel; \( \omega \) is the rotational speed; \( r \) is the effective rolling radius; \( F_x \) is the longitudinal ground force applied on the wheel; \( T_b \) and \( T_d \) are braking force and driving force, respectively; \( v_{\text{sw}} \) is the longitudinal speed at the wheel center.

In this paper, we only consider braking conditions, so \( T_d \) is set to be 0.

Design of the SMC Algorithm

The purpose of the SMC controller is to track the slip ratio of each wheel. In this case, we define the sliding surface as:

\[ s = S_L - S_N \]  

(44)

where, \( S_N \) is the reference slip ratio obtained from the previous section. The braking force on the sliding surface can be derived from the derivation of Eq. (44) by setting \( \dot{s} = 0 \):

\[ T_{b,s} = F_x (\mu_L - f) r \]  

(45)

where, \( \mu_L \) is assumed to be known. When the system is not on the sliding surface, the reaching law should satisfy the reachability condition as follows:

\[ s \dot{s} < -\eta |s|, \eta > 0 \]  

(46)

In this paper, a power-rate reaching law is used:

\[ \dot{s} = k |s|^\alpha \text{sign}(s) \]  

(47)

where, \( k \) and \( \alpha \) are carefully chosen. In order to gain better control effect of the SMC controller, two different reaching laws are designed on two sides of sliding surface, \( k \) and \( \alpha \) are set to be different values: \( k_p \) and \( \alpha_p \) for \( s \geq 0 \) and \( k_n \) and \( \alpha_n \) for \( s < 0 \).

SIMULATION AND RESULTS

Configuration of Simulation Condition

The controller is verified in Simulink-CarSim on both high-\( \mu \) (\( \mu = 0.8 \)) and low-\( \mu \) (\( \mu = 0.4 \)) roads and a typical D-class SUV is chosen in the simulation. Virtual sensors can be implemented easily in CarSim model. The yaw rate of the vehicle, vehicle side slip angle and slip ratios of wheels are obtained directly from CarSim to verify the control performance. The controller proposed in this paper is realized by using MATLAB embedded function and s-function. Basic parameters of the vehicle are presented in Table 1:

Table 1. Test vehicle configuration

<table>
<thead>
<tr>
<th>Vehicle parameters</th>
<th>Notations</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle mass</td>
<td>( m )</td>
<td>1609 kg</td>
<td></td>
</tr>
<tr>
<td>C.G. to the front</td>
<td>( a )</td>
<td>1.05 m</td>
<td></td>
</tr>
<tr>
<td>C.G. to the rear axle</td>
<td>( b )</td>
<td>1.569 m</td>
<td></td>
</tr>
<tr>
<td>Track width of front</td>
<td>( d_f )</td>
<td>1.565 m</td>
<td></td>
</tr>
<tr>
<td>Track width of rear</td>
<td>( d_r )</td>
<td>1.565 m</td>
<td></td>
</tr>
<tr>
<td>Vehicle yaw inertia</td>
<td>( I_{22} )</td>
<td>1765 kg m²</td>
<td></td>
</tr>
<tr>
<td>Tire type</td>
<td>/</td>
<td>185/65 R17</td>
<td></td>
</tr>
<tr>
<td>Effective rolling</td>
<td>( R_t )</td>
<td>0.357 m</td>
<td></td>
</tr>
</tbody>
</table>

In order to verify the control effect under extreme conditions, open-loop step input of the front steering wheel is employed. The steering angle is changed from 0 degree to 30, 40, 50 degrees in 0.2 second, respectively. Also, a growing pulse input test is conducted. The results are compared with those without control and those with only active front steering control (Controller B) with same inputs and road conditions. The vehicle entering speed is 120km/h. The configurations of the controller proposed in this paper (Controller A) are as follows:

MPC Controller:

Sampling time: \( T = 0.001 \)s

Predictive horizon: \( H_p = 10 \)

Control horizon: \( H_c = 2 \)

Weighting matrix for the target errors:

\[ Q = \begin{bmatrix} 10^4 & 0 \\ 0 & 5 \times 10^{13} \end{bmatrix} \]

Weighting matrix for the control inputs:

\[ R = \begin{bmatrix} 10^2 & 0 \\ 0 & 2.5 \times 10^1 \end{bmatrix} \]

Robust Allocation Algorithm:

\[ H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \]

\( S_{\text{min}} = 0.003 \); \( S_{\text{max}} = 1 \); \( dS_{\text{min}} = -0.3 \); \( dS_{\text{max}} = 0.3 \).
SMC Controller:

\[ k_p = 7500; \alpha_p = 0.77; k_n = 21500; \alpha_n = 0.91. \]

**Step Steering Input**

The step input of the front wheel is shown in Figure 6.

![Figure 6. Hand wheel step steer input](image)

**High Coefficient of Friction**

![Figure 7. Simulation results on high-\(\mu\) road: (a) yaw rate, \(\mu=0.8\), 30° step input; (b) slip ratios when controller A is used, \(\mu=0.8\), 30° step input; (c) yaw rate, \(\mu=0.8\), 40° step input; (d) slip ratios when controller A is used, \(\mu=0.8\), 40° step input; (e) yaw rate, \(\mu=0.8\), 50° step input; (f) slip ratios when controller A is used, \(\mu=0.8\), 50° step input.](image)
As shown in Figure 7, the yaw rates and wheel slip ratios under different step inputs are presented. For vehicle without controller, the overshoot and stable time of the yaw rate are very significant and increase evidently with the growing of the step input. By contrast, the controller A and controller B can both control the vehicle with small overshoot and stable time of the yaw rate. The overshoots of controller A under 30, 40 and 50 degrees step inputs are 0.85%, 1.30% and 1.51%, respectively. In contrast, the overshoots of controller B are 7.83%, 7.31% and 6.84%. What's more, owing to the robust optimal braking allocation algorithm, controller A can follow the reference yaw rate with less error than that of controller B under different step steering angle inputs.

Low Coefficient of Friction

Figure 8 shows that controller A can guarantee the tracing accuracy of yaw rate consistently well on low-μ road and the tracking error is much less than that of controller B. The overshoots of controller A under 30, 40 and 50 degrees step inputs are 0.10%, 1.27% and 2.46%, respectively. The overshoots of controller B are 5.70%, 7.65% and 10.28%. In contrast, the vehicle without controller has lost control. Moreover, tracking accuracies of slip ratios are also satisfying.
The target slip ratios and actual slip ratios of each wheel are shown in Figure 7 and Figure 8. In the figures, dash lines are target slip ratios and solid lines are actual slip ratios. There are three key points that are noteworthy:

1. In order to obtain the nominal additional yaw moment, the slip ratios of two wheels on the same side calculated from the robust optimal allocation algorithm are higher than that of the wheels on the other side, which can reserve the most tire force potential of each wheel. Thus, the results have verified the allocation effect of optimal robust allocation algorithm;

2. The actual slip ratio can follow the target slip ratio with acceptable precision and quick tracking speed. The relatively large tracking errors in certain parts of the curve are caused by the turning condition of the vehicle, which makes it hard to guarantee the tracking accuracy;

3. After the vehicle is stabilized after about 1.5s, the additional yaw moment is mainly decided by the tracking error of the vehicle side slip angle, which can explain that in Figure 7(b) (d), the high slip-ratio wheel is different from others after stabilization: the relative difference of vehicle yaw rate tracking error and vehicle side slip tracking error can decide the sign of additional yaw moment, which would further influence which wheels to brake in lower level of the controller.

**Growing Pulse Steering Input**

To further verify the control effect of the controller, a growing pulse input test is conducted on high-μ (μ=0.8) road, as shown in Figure 9:

The results suggest that without control, the vehicle can hardly track the desired yaw rate because of its large overshoot and stable time. Both controller A and controller B have satisfying control effects. Among them, Controller A can provide better control performance with less overshoot and stable time. Moreover, two important conclusions can be made:

1. As shown in Figure 11(b), the SMC controller can track the desired slip ratios with little error. The comparison between the curves in Figure 11(b) and Figure 11(d) indicates that the output of the robust optimal allocation algorithm is reasonable: the wheels with high slip ratios are on the inner side of the vehicle and can generate the desired additional yaw moment obtained from the MPC controller;
2. By comparing the curves in Figure 11(a), Figure 11(c) and Figure 11(d), it is verified that the output of the MPC controller is satisfying: the directions of both the additional yaw moment and additional steering angle are aimed at reducing the overshoot and the fluctuation of the yaw rate.

Robustness Verification

In order to verify the robustness of the controller, the inaccurate parameters of the vehicle are used instead of the nominal ones. To simulate the usage of vehicles in daily life, the center of gravity (CG) of the vehicle is changed by 20 percent to validate the robustness of the controller. Also, the total mass of the vehicle will not be constant in different load conditions and it should also be taken into consideration when verifying the robustness of the controller. The 50-degree step input of the front wheel is used as the input for a vehicle on low-μ road (μ=0.4):

Future researches will be focused on the following two areas. First, although the proposed controller in this paper has achieved satisfying control performance because of the rolling optimization process, its robustness can be further enhanced by using robust model predictive control (RMPC). Second, the relationship between the inaccuracies of the vehicle parameters and the uncertainties of the elements in the efficiency matrix $A$ can be further determined.

REFERENCES


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